Stochastic returns and open loop reverse logistic supply chain management

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Abstract. This paper deals with the issue of production control in reverse logistic supply chain with stochastic returns of end of life aluminum products. The production unit is in charge of recycling returned aluminum products and spring of aluminum ingots. It's considered as a machine producing one type of product which is subject to random breakdowns and repairs. The objective of this work is to find both optimal production and disposal policies of the system to minimize the global manufacturing costs over an infinite horizon. The model is developed using homogeneous Markov chains and stochastic dynamic programming approach to obtain the optimality conditions described by Hamilton-Jacobi equations. After numerical solution, we obtained an optimal control described by a modified hedging point policy. For the disposal, the optimal policy states that it should be done only when a sufficient stock of returned products required for production is available.

Keyword: Reverse logistic, Open loop supply chain, Stochastic dynamic programming, Numerical methods.

1. Introduction

Going on the basis that it's better to recapture the value of end of life products than dispose them (Rogers and Tibben-Lembke, 1998), many authors studied the opportunity to give a second life to end of life products (Kouedeu et al. (2014). For example, Kenné et al. (2012) studied the pproduction planning of a hybrid manufacturing–remanufacturing system by proposing a production policy that would minimize the costs. Based on the model presented in Kenné et al, (2012), this paper addresses the problem of optimal control in reverse logistic system for an open loop supply chain with random returns of end of life products. The rest of this paper is organized as follows. In Section 2, the description and context of the study are presented. In section 3, the problem statement is described. In section 4, a numerical approach to solve the optimality equations is presented. In section 5, a numerical example and results analysis are presented. Section 6 is reserved to discussion and suggestions for further researches.

2. Description and context of the study

We consider in this study an open loop reverse logistic supply chain with a recycling system assimilated to a single machine (M) subject to random breakdowns and repairs. The recycling system is supplied by a variable return of end-of-life aluminum products at variable rate r_1 or r_2 so as to constitute the stock level x_1. This collected stock can be disposed from the unit at a rate u_d or can be recycled at a rate u so as to establish the stock x_2 of finished products to meet the demand with constant rate d as shown in figure 1.

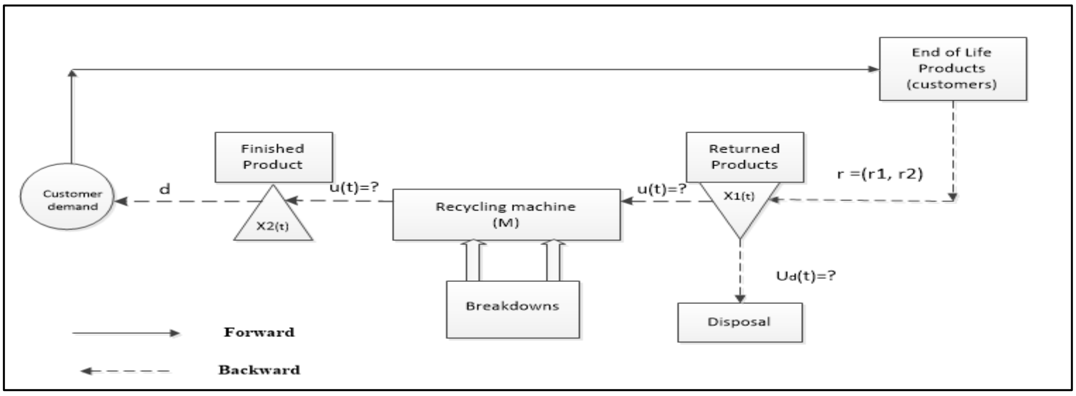

Figure 1. *Flow Material in the recycling unit*

3. Problem statement

We used the following notations to build the model

r_i (.) : value of the return rate at level i, $(i = 1,\ 2)$.

u_{d_α} (.) : Disposal rate of returned products at mode α $(\alpha = 1,\ 2,\ 3,\ 4)$

u_α (.) : Production rate of the machine at mode α (α =1,2).

$u_{\max}$: Maximal production rate of the machine.

d : Customer demand rate.

$x_1(t)$: Stock level of returned products at time t.

$x_2(t)$: Stock level of recycled products at time t.

$\alpha_1(t)$: Stochastic process of the machine.

$\alpha_2(t)$: Stochastic process of return rate.

c_1 : Inventory holding cost for returned products.

c_2^+ : Inventory holding cost for recycled products.

c_d : Disposal cost for returned products.

c_m : Cost for recycling process.

c_2^- : Backlog cost for recycled products.

c^α : Repair cost for the machine.

c_{env} : Environmental cost.

c_{ecart} : Cost penalizing the lack of returned products.

Before the problem formulation, it’s important to make some assumptions used in this paper. The main assumptions that support the model developed in this work are:

- The process is a homogeneous Markov chain process.
- The machine is prone to random breakdowns and repairs.
- The demand rate of the customers is constant and known.
- The maximum production rate of the system is constant and known.
- The return rates and the transition rates between them are known.
- When the machine breaks down, a corrective maintenance activity is immediately implemented.

Let $\alpha_1(t)=\{0,\ 1\}$ the stochastic process that describes the availability of the machine taking the value 0 or 1 depending on whether the machine is available or broken. Let also $\alpha_2(t)=\{1,\ 2\}$ the stochastic process describing the return rate of end-of-life products taking the value 1 or 2 depending on whether the return rate is r_1 or r_2. Then, $\alpha(t)=\alpha_1(t)\times\alpha_2(t)$ is the stochastic process which describes the state of the system. It takes values in $B=\{1,\ 2,\ 3,\ 4\}$ as shown by figure 2. For the global system, the 4×4 corresponding matrix of transition rates $Q=\left[\lambda_{ij}\right]$ is an ergotic process, which λ_{ij} respecting the following conditions:

$$\lambda_{ij}\geq 0\ \forall\ i\neq j \text{ and } \lambda_{ii}=-\sum_{i\neq j}\lambda_{ij} \tag{1}$$

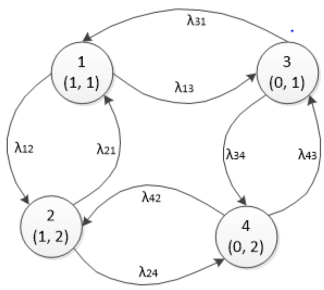

Figure 2: *States transition diagram of the system*

The set $\Gamma(\alpha)$ of the feasible control policies including the variables $u_{d_\alpha}(.)$ and $u_\alpha(.)$ is defined as follow:

$$\Gamma(\alpha)=\left\{(u_\alpha,\ u_{d_\alpha})\in R^2 / 0\leq u_\alpha\leq u_{\max}\ , 0\leq u_{d_\alpha}\leq r,\ \alpha=1,2,3,4\right\} \tag{2}$$

The recycling system will be able to meet the demand of customers over an infinite horizon, only if:

$$(\pi_1\times u_{\max}+\pi_2\times r_2) > \mathrm{d} \tag{3}$$

where π_i (i =1, 2, 3, 4) represents the limiting probability of the system at mode i. The reader is referred to the book of Ross (2007) for more details about limiting probabilities. Let g(.) be the cost rate defined as follow:

$$\begin{aligned} g(x_1,x_2,\alpha)=&\ c_1 x_1+(c_2^+x_2^++c_2^-x_2^-)+c_d u_{d_\alpha}+c_m u_\alpha+c^\alpha(ind\{\alpha(\mathrm{t})=3\}+ind\{\alpha(\mathrm{t})=4\}) \\ &+c_{env}(\mathrm{x}_{1\max}-\mathrm{x}_1)+\mathrm{c}_{\mathrm{ecart}}\left|(|\mathrm{x}_2|-\mathrm{x}_1)\right| ind\{\mathrm{x}_2<0\} \qquad \forall\alpha\in B \end{aligned} \tag{4}$$

where $x_2^+=\max(0,x_2)$; $x_2^-=\max(-x_2,0)$ and $ind\{\Theta(.)\}=\begin{cases}1, & if\ \ \Theta(.)\ is\ true\\ 0, & otherwise\end{cases}$

The total cost is given by:

$$J(x_1, x_2, \alpha) = \mathrm{E}\left\{ \int_0^\infty e^{-\rho t} g(x_1, x_2, \alpha)\,\mathrm{dt} \,\middle|\, x_1(0) = x_{10}, x_2(0) = x_{20}, \alpha(0) = \alpha_0 \right\} \tag{5}$$

where ρ is the discount rate. Our objective is to control the production rate u_α (.) and the disposal rate u_{d_α} (.) so as to minimize the expected discounted cost given by (5). The value function of such a problem is defined by :

$$v(x_1, x_2, \alpha) = \inf_{(u_\alpha,\ u_{d\alpha}) \in \Gamma(\alpha)} J(x_1, x_2, \alpha), \quad \forall \alpha \in \mathrm{B} \tag{6}$$

The value function $v(\cdot)$ satisfies the Hamilton–Jacobi–Bellman (HJB) equations (7) which describes the optimality conditions. More details on HJB equations can be found in Gershwin (1994).

$$\rho v(x_1, x_2, \alpha) = \min_{(u, u_d) \in \Gamma(\alpha)} \left[g(x_1, x_2, \alpha) + \sum_{\beta \in B} q_{\alpha\beta} v(x_1, x_2, \beta) + (r_\alpha - u_{d_\alpha} - u_\alpha) \frac{\partial v(x_1, x_2, \alpha)}{\partial x_1} + (u_\alpha - d) \frac{\partial v(x_1, x_2, \alpha)}{\partial x_2} \right] \tag{7}$$

The next section shows the numerical approach used to solve the HJB equations (7).

4. Numerical approach

The right side of HJB equation is minimized when the optimal control policy $(u_\alpha^*,\ u_{d_\alpha}^*) \in \Gamma(\alpha)$ is applied. It's now impossible to obtain analytically the solution of HJB equations (7). A numerical approach based on Kushner and Dupuis (1992) method is used to obtain a sub-optimal solution. After several manipulations as appeared in the works of Kenne et al. (2003), the HJB equations (7) become:

$$v^h(x_1, x_2, \alpha) = \min_{(u_\alpha, u_{d_\alpha}) \in \Gamma(\alpha)} \left[\begin{array}{l} \left(\rho + |\lambda_{\alpha\alpha}| + \frac{|r_\alpha - u_{d_\alpha} - u_\alpha|}{h_1} + \frac{|u_\alpha - d|}{h_2}\right)^{-1} \times \\ \left\{ \begin{array}{l} c_1 x_1 + c_2^+ x_2^+ + c_2^- x_2^- + c_d u_d + c_m u_\alpha + c^\alpha (ind\{\xi(\mathrm{t})=3\} + ind\{\xi(\mathrm{t})=4\}) + c_{env}(\mathrm{x}_{1\max} - \mathrm{x}_1) + \mathrm{c}_{\mathrm{ecart}} \left| (|\mathrm{x}_2| - \mathrm{x}_1) \right| ind\{\mathrm{x}_2 < 0\} + \sum_{\beta \neq \alpha} \lambda_{\alpha\beta} v^h(x_1, x_2, \beta) \\ + \frac{|r_\alpha - u_{d_\alpha} - u_\alpha|}{h_1} v^{h_1}(x_1 + h_1, x_2, \alpha) ind\{r_\alpha - u_{d_\alpha} - u_\alpha \geq 0\} + \frac{|r_\alpha - u_{d_\alpha} - u_\alpha|}{h_1} v^{h_1}(x_1 - h_1, x_2, \alpha) ind\{r_\alpha - u_{d_\alpha} - u_\alpha < 0\} \\ + \frac{|u_\alpha - d|}{h_2} v^{h_2}(x_1, x_2 + h_2, \alpha) ind\{u_\alpha - d \geq 0\} + \frac{|u_\alpha - d|}{h_2} v^{h_2}(x_1, x_2 - h_2, \alpha) ind\{u_\alpha - d < 0\} \end{array} \right\} \end{array} \right] \tag{8}$$

5. Analysis of results

In this section, we present the results obtained by solving numerically the HJB equations (7). The parameters used are given in table 1 below.

Table 1: *Parameters of the numerical example (UT: unit of time)*

c_m	c_d	c_1	c_{2p}	c_{2m}	c_{alpha}	c_{ecart}	C_{env}	ρ	r	d	u_{max}	q12	q21
14	1.5	1.8	3.5	55	13	55	1	0.1	0.55	0.4	0.5	0.01	0.1
$/product/UT	$/product/UT	$/product/UT	$/product/UT	$/product/UT	$/breakdown/UT	$/product/UT	$/product/UT		product/UT	product/UT	product/UT	/UT	/UT

Equations (9) and (10) resume the optimal production policy at mode 1 (illustrated by figure 3) and the disposal policy at the same mode (illustrated by figure 4) respectively.

$$u(x_1, x_2, 1) = \begin{cases} u_{\max} & if \;\; x_2 < \psi(x_1) \\ d & if \;\; x_2 = \psi(x_1) \\ 0 & if \;\; x_2 > \psi(x_1) \end{cases} \qquad \textbf{(9)}$$

$$u_d(x_1, x_2, 1) = \begin{cases} r_1 & if \;\; x_1 > \sigma(x_2) \\ r_1 - u_{\max} & if \;\; x_1 = \sigma(x_2) \\ 0 & if \;\; x_1 < \sigma(x_2) \end{cases} \qquad \textbf{(10)}$$

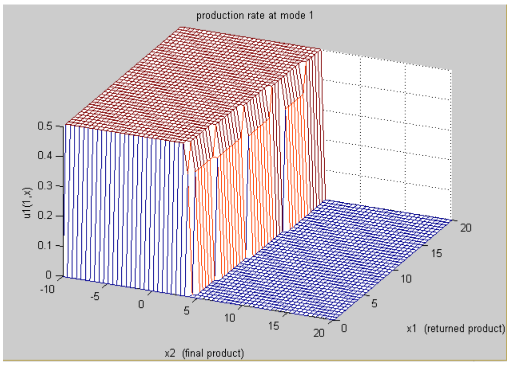

Figure 3: Production rate at mode 1

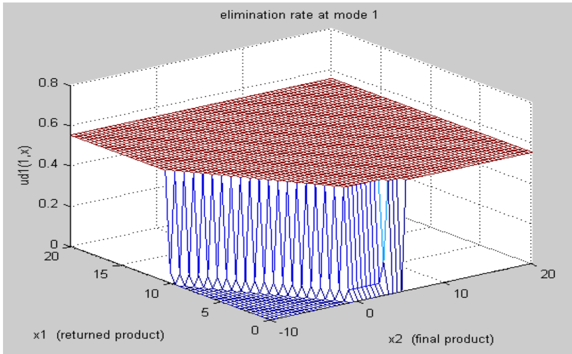

Figure 4: Disposal rate at mode 1

The production policy states the following:

- Set the production rate of the machine to its maximal production rate when the stock level of the final products x2 is under the threshold $z_{21} = \psi(x_1)$, where $\psi(x_1)$ is a function depending on the level of returned product x1.
- Set the machine rate to the demand rate d when the level of stock x_2 is equal to the threshold $z_{21} = \psi(x_1)$.
- Stop the production of the machine when the level of stock x2 is greater than the threshold $z_{21} = \psi(x_1)$.

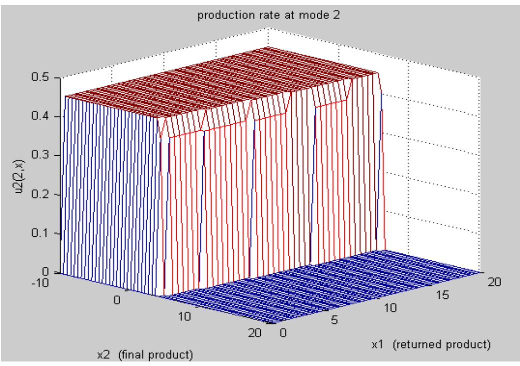

Figure 5 Production rate at mode 2

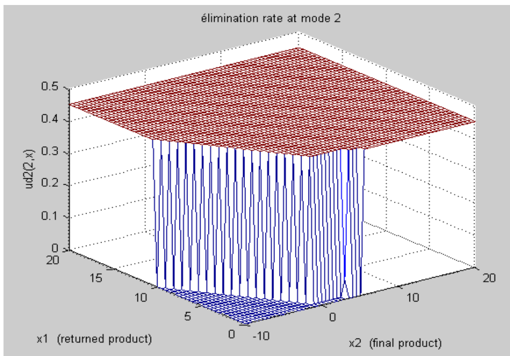

Figure 6 Disposal rate at mode 2

The disposal policy described by equation (10) can be explained in the same way in which $z_{11} = \sigma(x_2)$ is the threshold for the disposal rate depending on the level of the final products. The hedging point policy obtained is an extension of woks of Akella and Kumar (1986). Although they have obtained a fixed threshold, we obtained a variable threshold levels. In the same way, production and disposal policies at mode 2 are given in figure 5 and 6 and can be resumed by equations (11) and (12).

Results obtained are realistic because, during the production at mode 1 and 2, returned products are sufficiently stored so as to supply the need of production by

$$u(x_1,x_2,2)=\begin{cases} r_2 & si \quad x_2<\varpi(x_1) \\ d & si \quad x_2=\varpi(x_1) \\ 0 & si \quad x_2>\varpi(x_1) \end{cases} \quad \textbf{(11)}$$

$$u_{d_2}(x_1,x_2,2)=\begin{cases} r_2 & si \quad x_1\geq\delta(x_2) \\ 0 & si \quad x_1<\delta(x_2) \end{cases} \quad \textbf{(12)}$$

minimising the holding and backlog cost. During the breakdown at mode 3 and 4, there is no production and the disposal policies are given by figures 7 and 8, resumed respectively by equation 13 and 14.

$$u_{d_3}(x_1,x_2,3)=\begin{cases} r_1 & si \quad x_1\geq\eta(x_2) \\ 0 & si \quad x_1<\eta(x_2) \end{cases} \quad \textbf{(13)}$$

$$u_{d_4}(x_1,x_2,4)=\begin{cases} r_2 & si \quad x_1\geq\iota(x_2) \\ 0 & si \quad x_1<\iota(x_2) \end{cases} \quad \textbf{(14)}$$

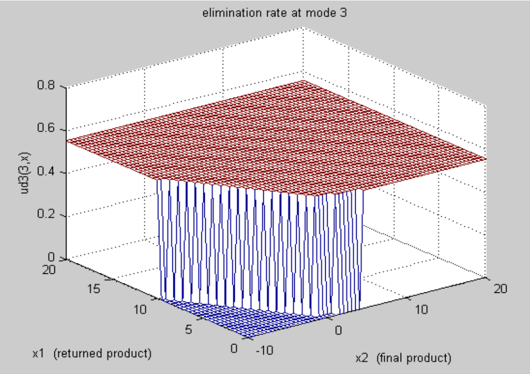

Figure 7 Disposal rate at mode 3

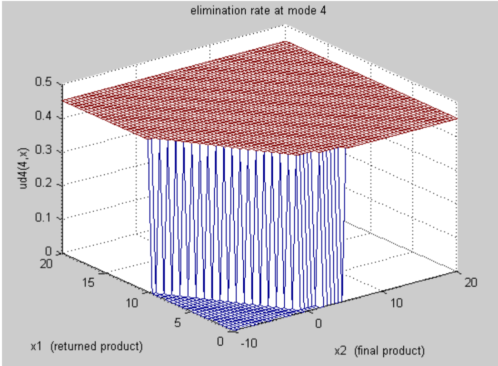

Figure 8 Disposal rate at mode 4

6. Conclusion

We can conclude that the goal has been achieved. We have determinate an optimal production and disposal policies for a stochastic open loop reverse logistic supply chain under specific assumptions. These results can help recycling industries in open loop reverse logistics to optimise their performance by minimising the total cost as defined in this paper. It might also be interesting to study this topic in further research by considering the different steps in the recycling process so as to assimilate the system by two tandem machines producing one type of products.

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Bericht zum 63. Arbeitswissenschaftlichen Kongress vom 15. – 17. Februar 2017

FHNW Brugg-Windisch, Schweiz

Herausgegeben von der Gesellschaft für Arbeitswissenschaft e.V.
Dortmund: GfA-Press, 2017
ISBN 978-3-936804-22-5

NE: Gesellschaft für Arbeitswissenschaft: Jahresdokumentation

Als Manuskript zusammengestellt. Diese Jahresdokumentation ist nur in der Geschäftsstelle erhältlich.

Schriftleitung: Matthias Jäger
im Auftrag der Gesellschaft für Arbeitswissenschaft e.V.

USB-Print: Dr. Philipp Baumann, Olten

Screen design und Umsetzung
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